On Macaulay Modules and Abstract Hilbert Polynomials

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Abstract

Let A denote a commutative noetherian ring with identity, M a finitely generated A-module, and $\underline{a}$ an ideal of A such that the module $M/\underline{a}M$ has finite length, $L(M/\underline{a}M)$, i.e. such that every prime ideal belonging to $\underline{a}M$ is maximal. Let $\chi(\underline{a},M;n+1) = \sum_{i=0}^{h} (-1)^i e_i(\underline{a},M)\binom{n+h-i}{h-i}$, where the $e_i(\underline{a},M)$ are integers, be the polynomial in n which, for sufficiently large values of n, coincides with $L(M/\underline{a}^{n+1}M)$. M is called a Macaulay module if for every maximal ideal $\underline{m}$ containing $\operatorname{ann} M = (0):M$ the following integers coincide: (i) altitude $(A/\operatorname{ann} M)$, (ii) the length of a maximal $M_{\underline{m}}$-sequence, and (iii) the degree of the polynomial $\chi(A_{\underline{m}},M_{\underline{m}};n+1)$. If this is the case, these integers also coincide with the degree h of $\chi(\underline{a},M;n+1)$. Theorem. Let M be a Macaulay module. If $h \geq 1$, $e_1(\underline{a},M) \geq e_0(\underline{a},M) - L(M/\underline{a}M) \geq 0$. If $h \geq 2$, $e_2(\underline{a},M) \geq 0$. Theorem. Let M be a Macaulay module with $h \geq 1$. Then the following are equivalent: (i) $\chi(\underline{a},M;n+1) = L(M/\underline{a}M)\binom{n+h}{h}$, (ii) $e_1(\underline{a},M) = 0$, (iii) $e_0(\underline{a},M) - L(M/\underline{a}M) = 0$, and are implied by: (iv) there exist $x_1, \ldots, x_h \in \underline{a}$ such that $\underline{a}M = (x_1, \ldots, x_h)M$. If, in addition, A is a local ring, then (i), (ii), or (iii) implies (iv).

An elementary development of Macaulay modules over local rings is used to extend results of Northcott (J. London Math. Soc., 35 (1960), 209-214) and Narita (Proc. Camb. Phil. Soc., 59 (1963), 269-275) concerning Macaulay local rings to Macaulay modules over rings which are not necessarily local. The theorems above are typical.

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